WRITTEN HOMEWORK #2, DUE JAN 20, 2010

You may turn this assignment into the homework boxes outside the classroom (Kemeny 008) or at the beginning of class. Remember that you need to provide correct details to receive full credit.

(1) (Chapter 15.6, #56) Show that every normal line to the sphere $x^2 + y^2 + z^2 = r^2$ passes through the origin.

(2) (Chapter 15.6, #61) (a) Two surfaces are called orthogonal at a point of intersection if their normal lines are orthogonal at that point. Show that if $F(x, y, z) = 0, G(x, y, z) = 0$ are two surfaces which are orthogonal at a point P where $\nabla F(P) \neq \mathbf{0}, \nabla G(P) \neq \mathbf{0}$, then

$$F_x G_x + F_y G_y + F_z G_z = 0$$

at P.

(b) Use (a) to show that the two surfaces $z^2 = x^2 + y^2, x^2 + y^2 + z^2 = r^2$ are orthogonal at every point of intersection. Can you see why this is true without using any calculus? (For this last part, you do not need to give a completely rigorous explanation.)

(3) (Based on Problems Plus #1, Chapter 16) Let $R = [0, 2] \times [0, 2]$, and let $[[x]]$ be the greatest integer less than or equal to x. For example, $[[2]] = 2, [[-1.5]] = -2$, and $[[\pi]] = 3$. Evaluate

$$\iint_R [[x]] + [[y]] \, dA.$$

(Hint: Try to graph $z = [[x]] + [[y]]$ over R and interpret the double integral as the volume of a certain region.)

(4) (Chapter 16.2, #12) Let $R = [0, 5] \times [0, 5]$. Evaluate the double integral

$$\iint_R (5 - x) \, dA$$

by (a) interpreting the value of the double integral as the volume of a solid, whose volume you compute from geometry, (b) evaluating an iterated integral with order of integration $dx\,dy$, and (c) evaluating an iterated integral with order of integration $dy\,dx$.

(5) (Chapter 16.3, #16) Let D be the region of the xy plane bounded by $x = 0, x = \sqrt{1 - y^2}$. Evaluate

$$\iint_D xy^2 \, dA.$$

(6) (Chapter 16.3, #44) Consider the iterated integral

$$\int_0^1 \int_{\arctan x}^{\pi/4} f(x, y) \, dy \, dx.$$

Sketch the region of integration represented by this iterated integral and write down the iterated integral obtained by interchanging the order of integration.

(7) (Chapter 16.3, #50) Evaluate the following integral by reversing the order of integration:

$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4}\,dx\,dy.$$

(8) (Chapter 16.3, #58) When evaluating a certain double integral over a region D, the following sum of iterated integrals was obtained:

$$\iint_D f(x,y)\,dA = \int_0^1 \int_0^{2y} f(x,y)\,dx\,dy + \int_1^3 \int_0^{3-y} f(x,y)\,dx\,dy.$$

Sketch the region D, and then write down the iterated integral obtained by interchanging the order of integration.